LINEAR WAVE MOTION FROM CONCENTRATED HARMONIC SOURCES IN BLASIUS FLOW

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Abstract

The motion of individual linear instability waves in shear flows is well described by existing theoretical and numerical methods. However, naturally occuring sources produce coherent wave motions with broadband spanwise-wavenumber and frequency spectra, and the different spectral components interact both linearly and nonlinearly. The simplest possible example of such wave motion is the wave pattern produced by a small-amplitude harmonic point source in a Blasius boundary layer. Even this example offers a challenge to both the theorist and experimenter. We attempt to meet this challenge numerically by a series of calculations for the parameters of three different experiments using locally-parallel linear stability theory (LST), the Parabolized Stability Equations (PSE) and Direct Numerical Simulation (DNS). The calculations illustrate the strengths and weaknesses of the different methods, the extent to which the methods agree or disagree, and, finally, the extent to which agreement with the measurements can be attained, given that the experiments also have their own difficulties. With the assumption of a uniform initial spanwise-wavenumber spectrum, there is little difference between the PSE wave pattern near the level of maximum amplitude and LST. In contrast to earlier conclusions, nonparallel effects are minimal. Comparisons with the measurements show a strong influence of the source. Some of this effect can be captured by use of a suitable initial spanwise-wavenumber spectrum, but the disagreement in the near-field and close to the centerline in the far-field can only be resolved by a high-resolution DNS code.

1 Introduction

Much of the existing work on unstable waves in shear flows is devoted to the study of either individual linear waves or the nonlinear interaction of only a few waves. These wave motions are well described by existing theoretical and numerical methods. However, naturally occuring sources do not often produce pure instability modes, even in two-dimensional (2D) shear flows, but rather wave motions with broad spectra of spanwise wavenumbers and frequencies. If the source is sufficiently concentrated, the wave motion is coherent, and the different spectral components interact linearly as well as nonlinearly. If the initial amplitude is small enough to eliminate nonlinearity, the subsequent wave motion can be obtained either by linear superposition of a great many modes, or by asymptotic methods, provided the initial spectra are known. The simplest possible example of such a linear wave motion is the wave pattern produced by a harmonic point source (HPS) in a Blasius boundary layer.

The first work on this problem was a well-documented, detailed experiment by Gilev, Kozlov & Kachanov.[1,2] The wave motion in this experiment was entirely linear. A more limited experiment, plus a calculation of the wave pattern, was carried out by Mack & Kendall.[3,4,5] The calculations used locally-parallel linear stability theory (LST) together with the assumption of a uniform initial spectrum. Although the phase calculations were in good agreement with the measurements, a large disagreement with the amplitude measurements was attributed to nonparallel effects, because the same correction for boundary-layer growth that Gaster[6] used to bring his calculations of the wave packet produced by an impulsive point source into agreement with the experiment of Gaster & Grant,[7] also brought the calculated centerline amplitude of the HPS wave pattern into agreement with the measurements. Balakumar & Malik[8]

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